(NASA-CR-154839) EFFECTS OF SOLAR AND LUNAR GRAVITY ON LIFETIME OF EARTH PARKING ORBITS (Bellcomm, Inc.) 33 p

N79-71956

Unclas
00/13 12510

BELI
1100 SE'

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Effects of Solar and Lunar Gravity on Lifetimes of Earth Parking Orbits

TM- 67-1013-5

DATE- August 9, 1967

FILING CASE NO(S)- 103-2

AUTHOR(S)- H. B. Bosch

FILING SUBJECT(S)- Orbital Perturbations
(ASSIGNED BY AUTHOR(S)- Parking Orbit Lifetime

ABSTRACT

The perturbing effects of lunar and solar gravity, as well as earth oblateness, on the elements of elliptic earth orbits are examined. A detailed study is made of these effects on parking orbits associated with a nominal trajectory for a proposed 1975 Mars flyby mission. For comparison, summary results are presented for three other proposed planetary missions, i.e., a Venus flyby in 1975, one in 1977, and a triple-planet mission in 1976.

It is shown that solar and lunar perturbations can be severe and that injection parameters must, in general, be selected with care if significant lifetimes are expected of the parking orbit. However, it is concluded that parking orbits with reasonably constant perigee altitudes exist for the two Venus missions. The ΔV requirements for maintaining perigee altitude for the other two missions are estimated to be only a small portion of the total mission requirements.

SEE REVERSE SIDE FOR DISTRIBUTION LIST

DISTRIBUTION

COMPLETE MEMORANDUM TO

CORRESPONDENCE FILES:

OFFICIAL FILE COPY
plus one white copy for each
additional case referenced

TECHNICAL LIBRARY (4)

NASA Headquarters

P. E. Culbertson/MLA
J. H. Disher/MLD
F. P. Dixon/MTY
E. W. Hall/MTS
T. A. Keegan/MA-2
D. R. Lord/MTD
M. J. Raffensperger/MTE
L. Reiffel/MA-6
A. D. Schnyer/MTV
G. S. Trimble/MT

MSC

M. A. Silveira/ET25
W. E. Stoney, Jr./ET
J. M. West/AD

MSFC

R. J. Harris/R-AS/VP
B. G. Noblitt/R-AERO-DPF
F. L. Williams/R-AS-DIR

KSC

J. P. Claybourne/EDV4
R. C. Hock/PPR2
N. P. Salvail/MC

COVER SHEET ONLY TO

Bellcomm, Inc.

F. G. Allen
G. M. Anderson
A. P. Boysen
J. O. Cappellari, Jr.
C. L. Davis
J. P. Downs
D. R. Hagner
P. L. Havenstein
J. J. Hibbert
W. C. Hittinger
B. T. Howard
D. B. James
H. S. London
K. E. Martersteck
R. K. McFarland
J. Z. Menard
I. D. Nehama
G. T. Orrok
C. A. Pate
I. M. Ross
R. L. Selden
R. V. Sperry
J. M. Tschirgi
R. L. Wagner
J. E. Waldo
All Members, Division 101
Department 1023

BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D. C. 20036

SUBJECT: Effects of Solar and Lunar Gravity on Lifetimes of Earth Parking Orbits
Case 103-2

DATE: August 9, 1967

FROM: H. B. Bosch

TM-67-1013-5

TECHNICAL MEMORANDUM

1.0 INTRODUCTION

A proposed method of departing on an interplanetary mission is to place stages of the space vehicle into an elliptic parking orbit around earth by separate launches and then to assemble the total vehicle in this orbit. An impulsive thrust applied at perigee will then start the interplanetary journey. Such an assembly operation may require the pieces--separately or assembled--to remain in the parking orbit for as long as six months.

Several launches from earth will be required to place the stages into the parking orbit. The injection maneuver here is considered to consist of a Hohmann transfer from a 100 nautical mile circular orbit to a circular orbit of altitude h_p, then a single impulse to change the circular orbit into an ellipse with perigee altitude h_p. The velocity requirement for this injection maneuver may be expressed symbolically as*

$$\Delta V_i = \Delta V_H + (V_p - V_c) \tag{1}$$

This injection velocity requirement is plotted in Figure 1 as it varies with perigee altitude and orbital period of the resulting parking orbit.

At the other end of the assembly time period it will be necessary to transfer from the parking orbit to the interplanetary departure trajectory. The velocity requirement for a one-impulse transfer is

[redacted]

$$\Delta V_L = V_L - V_p \tag{2}$$

---

*For definition of all symbols see List of Symbols.

The dependence of this velocity difference on perigee altitude and orbital period is shown in Figure 2.

Although the numerical value for V_∞ used in Figure 2 is that for a 1975 Mars flyby mission it is the effect of h_p on these curves that is significant: low perigee altitudes are desirable at injection as well as at departure. In this memorandum perigee altitudes between 100 and 300 nautical miles and orbital periods of one to two days will be considered as typical.

This means that such parking orbits can have apogee radii from 42,000 to 70,000 nautical miles. It is then to be expected that the gravity of the moon and the sun might have a significant effect on the orbital elements, especially in view of the possibly long (up to six months) lifetimes expected of such orbits.

Before studying these perturbations, a discussion of some geometric requirements for such parking orbits is in order. Following that, the effects of the moon and sun on perigee histories are examined qualitatively. The results of a perturbation analysis of the family of parking orbits for a Mars flyby mission are presented in detail to show the effects of orbital inclination and period on orbit lifetime. In this context typical ΔV requirements for maintaining perigee altitude are also shown. Finally, similar results for three other proposed planetary missions are presented in summary. It is assumed that transfer to the interplanetary leg of the trip can be made from perigee with a single in-plane impulse; i.e., parking orbits which require an inclination change or reorientation prior to departure are not considered in this memorandum.

2.0 PARKING ORBIT PARAMETRIZATION

2.1 The Orbital Plane

An interplanetary trajectory can be described by a geocentric hyperbolic arc which is tangent to a heliocentric elliptic arc at the earth's so-called sphere of influence. The geocentric escape hyperbola is characterized by its asymptote, whose direction is taken to be parallel to the common tangent of the two arcs. This direction is called the departure asymptote and will be represented vectorially by:

$$\vec{V}_\infty = (\alpha_\infty, \delta_\infty, V_\infty) \quad .$$

Thus the primary geometric constraint on an earth parking orbit for a given interplanetary trajectory is that its orbital plane contain the departure asymptote. This defines an infinity of orbital planes. If we use the orbital inclination (i) as parameter, each value of i then defines two of these planes as shown in Figure 3. Excluding retrograde orbits we see that, for each i, one orbit will be on the ascent as it passes through the departure asymptote and the other on the descent. We will refer to the former family of orbits as "type 1" and to the latter as "type 2." Specifically, referring to Figure 4, the ascending node, Ω, is obtained from

$$\sin (\alpha_\infty - \Omega) = \tan \delta_\infty / \tan i \qquad (3)$$

Choosing the value of Ω for which $0^\circ \leq \alpha_\infty - \Omega < 90^\circ$ results in a type 1 orbit. A type 2 orbit results if $90^\circ < \alpha_\infty - \Omega \leq 180^\circ$.

There is a unique orbit between these two types: an orbit which neither ascends nor descends as it passes through $\vec{V}_\infty$ and for which $i = \delta_\infty$. This will be called the "minimum inclination orbit" because no parking orbit (as defined in this context) can have an inclination less than the declination of the departure asymptote.

2.2 In-Plane Orbital Elements

Once the orbital plane has been fixed the location of perigee relative to $\vec{V}_\infty$ can immediately be determined from dynamical considerations by the equation*

$$\cos \theta_\infty = \frac{-1}{1 + \frac{r_p}{\mu} V_\infty^2} \qquad (4)$$

where $90^\circ < \theta_\infty \leq 180^\circ$. This leads to the computation of ω, the

*This is valid if the transfer to the hyperbola is accomplished impulsively at perigee. See Figure 5.

argument of perigee, in the formula

$$\sin(\omega + \theta_\infty) = \sin \delta_\infty / \sin i \qquad (5)$$

As before, a type 1 or type 2 orbit results according as $0^\circ \leq \omega + \theta_\infty < 90^\circ$ or $90^\circ < \omega + \theta_\infty \leq 180^\circ$ is chosen.

Finally, the eccentricity is given by

$$e = 1 - r_p/a \qquad (6)$$

where a is related to the orbital period by

$$a = [\mu (P/2\pi)^2]^{1/3} \qquad (7)$$

Thus, for this method of parametrization, the inclination defines the orbital plane (Equation 3); the perigee altitude fixes the orientation of the ellipse in its plane (Equations 4 and 5); and the orbital period determines the shape of the ellipse (Equations 7 and 6).

3.0 ORBITAL PERTURBATIONS

3.1 Sources of Perturbation

The time history of the parking orbit can be adequately described as that of a Keplerian ellipse whose orbital elements are perturbed during each orbit by the gravitational pull of the moon and sun, as well as the oblateness of the earth. The effects of atmospheric drag will not be considered in this memorandum.

The primary objective of this study is to examine those perturbations which may affect the lifetime of the orbit. Therefore, the perigee altitude during the required time span prior to the interplanetary departure date will be the crucial variable. In the following four sections the perturbation sources will be discussed qualitatively.

3.2 Lunar Perturbations

An examination of the equations describing the perigee radius perturbations per orbit due to the gravitational pull of a third body shows that the motion has a periodic component and a secular component (e.g., References 1, 2, 3, and 4). Specifically, the perigee altitude perturbation per orbit can be written as (Reference 1)*

$$\Delta h_p = KP^2 ae \sqrt{1-e^2} [(\cos 2\omega_r \cos 2i_r) \sin 2\gamma - (\sin 2\omega_r) \cos 2\gamma - (\sin^2 i_r \sin 2\omega_r) \sin^2 \gamma] \quad (8)$$

where the constant K is a characteristic of the disturbing body. Thus, if the argument of perigee and inclination are held fixed relative to the orbit of the disturbing body (see Figure 6), the first two terms are periodic and integrate (with respect to γ) to zero over one orbit of the disturbing body. The last term, however, is the secular component.

In the case of the moon, the periodic component has a period of one-half month and the sign of the secular component depends on ω_r, or on the orientation of the line of apsides of the orbit relative to the orbit of the moon. This is illustrated by the example in Figure 7 where the two-week lunar oscillations are superposed on a secular downward trend.

3.3 Solar Perturbations

The contribution of solar perturbations can also be described by Equation 8 except that the magnitudes of K, ω_r, and i_r will be different. Here, however, the oscillatory terms have a period of six months. The example in Figure 7 shows how the solar oscillatory term dominates the motion. The secular component is indiscernible over this three month time span. Thus, the combined lunar and solar perturbations result in the indicated time history of perigee altitude.

---

*The perturbation model presented here is based on the assumption that the disturbing body (Moon and Sun) moves uniformly in a circular orbit and that its position can be considered fixed during one revolution of the vehicle in the parking orbit. Both assumptions are reasonable ones, especially in the context of this memorandum.

3.4 Oblateness Perturbations

When the earth is considered to be an ellipsoid of revolution (i.e., having a circular equator rather than a "bumpy" equator) this asphericity affects only the nodal and apsidal lines. Specifically, the nodal and apsidal precessions per orbit are given by (Reference 5):

$$\Delta\Omega = -2\pi\ J\ \cos\ i/[(a/R)(1-e^2)]^2 \tag{9}$$

$$\Delta\omega = \pi\ J(4-5\ \sin^2\ i)/[(a/R)(1-e^2)]^2 \tag{10}$$

Note that the nodes always regress, except for a polar orbit. The argument of perigee advances for inclinations $i < 63^{\circ}26'$ and regresses for $i > 63^{\circ}26'$. Thus, although perigee altitude is not perturbed directly by oblateness, the quantities ω_r and i_r in equation (8) are affected by oblateness through the precessions indicated by (9) and (10). For example, an orbit with a two-day period, 100 nautical mile perigee altitude, and $i = 33^{\circ}$, experiences nodal and apsidal precessions of -5.7° and $+8.6^{\circ}$, respectively, over a time span of three months.

3.5 Atmospheric Drag Perturbations

The dominant effect of air drag on an elliptic orbit is to decrease the major axis (Reference 6). Initially the apogee altitude is lowered while perigee altitude remains virtually unaffected. This shortens the orbital period and reduces eccentricity. When the eccentricity is "small enough" the perigee altitude is also lowered. However, in the case of the type of orbits to be considered in this context, these perturbations will most likely be dominated by solar and lunar effects. A quantitative analysis will depend on such factors as the aerodynamic characteristics of the particular vehicle, diurnal variations in the atmosphere, sunspot activity, etc. Therefore, atmospheric perturbations are not considered in this memorandum and the orbital period is considered constant in computations.

4.0 A 1975 MARS FLYBY MISSION

The departure asymptote for the Mars flyby mission departing earth on September 23, 1975, is given by (Reference 7):

$$\alpha_\infty = 85.7^\circ$$

$$\delta_\infty = 32.95^\circ$$

$$V_\infty = 0.1985 \text{ emos}$$

Thus the minimum inclination orbit will have $i = 32.95^\circ$, with the orientation (nodal line) of the plane of this and higher inclination orbits to be determined by Equation 3.

It can be shown from Equation 8 that, for orbital periods between one and two days, a difference in perigee altitude of 100 nautical miles will affect the perigee perturbation (Δh_p) by less than two percent. Thus taking h_p = 200 nautical miles as a nominal perigee altitude the resulting perigee histories can be considered as typical for the parking orbits which are feasible.

Computations were performed for orbits ranging from minimum inclination to polar orbits. All orbits were selected so as to attain h_p = 200 nautical miles at earth departure, as well as inclination, ascending node, and argument of perigee corresponding to the departure asymptote.*

Selected perigee altitude histories for the Mars mission are shown in Figure 8a. These are for parking orbits with two day periods. In order to make maximum use of the earth's rotation at launch, the orbital inclination should be as close as possible to the latitude of the launch site which, for the Kennedy Space Center, is approximately 28.5°. The most advantageous orbit in this regard is the minimum inclination orbit, indicated by $i = \delta_\infty$ on Figure 8a. Note that the perigee altitude for this orbit dips below 100 nautical miles before rising to 200 nautical miles. Such orbits are unrealistic as they stand because they cannot survive the severe atmospheric effects at these low altitudes. However, as was indicated before, they may be "shifted" upward by 100 or 200 nautical miles without significantly altering the shape of the corresponding perigee altitude curve. This would, of course, result in a higher injection altitude with

---

*Actually the appropriate equations were computed backward in time from the desired conditions on the departure date.

a corresponding ΔV penalty. The ΔV penalty for a higher perigee altitude at departure time is as shown in Figure 2.

Figure 8a also shows that higher inclination orbits (e.g., $i = 50^\circ$) can be found which never go below, say, 150 nautical miles. In particular, the two types of polar orbits always stay above 200 nautical miles.

The effects of orbital inclination on perigee history are shown in Figure 8a for a fixed orbital period. A further look at Equation 8 reveals that the amplitude of the oscillations depends on the period although the phase remains unaltered. Therefore, the same cases shown in Figure 8a are plotted in Figures 8b and 8c for periods of 1-1/2 and 1 day, respectively. Comparison of these three figures reveals that shorter orbital periods result in smaller amplitude lunar and solar oscillations, as expected. The qualitative aspects (e.g., rising, falling, in a "trough", etc.), however, are still present. Thus, for this Mars mission, the permissible parking orbits are all such that either (1) injection must occur at very high altitudes, or (2) intermittent thrusts must be applied to raise perigee.

An impulse applied at apogee to raise perigee by Δh_p can be represented by

$$\Delta V_a = \frac{1}{4} \sqrt{\frac{\mu}{a}} \sqrt{\frac{r_a}{r_p}} \frac{\Delta h_p}{a} \tag{11}$$

and a corresponding perigee impulse, to lower apogee by the same amount, by

$$\Delta V_p = \frac{1}{4} \sqrt{\frac{\mu}{a}} \sqrt{\frac{r_p}{r_a}} \frac{\Delta h_p}{a} \tag{12}$$

Choosing $\Delta h_p = 50$ nautical miles as typical, the total characteristic velocity required for such a two-impulse maneuver is plotted in Figure 9 as a function of orbital period. (Note the closeness of the two curves for $h_p = 100$ nautical miles and $h_p = 300$ nautical miles.) These results show that the ΔV requirements for maintaining perigee altitude may be kept to a very small portion of the overall launch-assembly-departure operation requirements.

The total changes which the angular elements (i, Ω, and ω) experience over an 84-day assembly period are shown in Figures 10a and 10b as a function of the orbital inclination at the time of departure. Although they do not directly affect orbital lifetime, Δi and $\Delta\Omega$ are shown here because the initial orbital plane has to be biased appropriately; and $\Delta\omega$ determines the phase of the circular orbit at which an impulse must be applied to inject into the elliptic parking orbit. The respective ranges of the perturbations can be represented approximately by

$$-3.9^\circ < \Delta i < 1.7^\circ$$

$$-12^\circ < \Delta\Omega < 0^\circ$$

$$-7^\circ < \Delta\omega < 19^\circ$$

As a contributor to $\Delta\Omega$ and $\Delta\omega$, earth oblateness generally dominates over lunar and solar effects by one or two orders of magnitude although this relationship varies with orbital inclination. It should be noted here that the effects of oblateness diminish as the orbital period increases (see equations 9, 10, and 7) whereas lunar and solar perturbations become more severe.

5.0 THREE OTHER INTERPLANETARY MISSIONS

5.1 A 1975 Venus Flyby Mission

The departure asymptote for the proposed Venus lightside flyby mission departing earth on June 7, 1975, is given by (Reference 7)

$$\alpha_\infty = 144.55^\circ$$

$$\delta_\infty = -17.68^\circ$$

$$V_\infty = 0.1087 \text{ emos}$$

Some representative perigee histories for this mission are shown in Figure 11.

Without thrusting maneuvers the minimum inclination orbit (again indicated by $i = \delta_\infty$) is clearly unsuitable for injection prior to about 20 days before departure; so is the type 1 orbit with $i = 30^\circ$ for injection prior to about 40 days before departure. (The type 2 curve for $i = 30^\circ$ is not shown in Figure 11 because it is almost identical to the minimum inclination curve.)

On the other hand one could inject into a low inclination orbit (say, $i = 30^\circ$) at low altitude and then pay the ΔV penalty associated with a high departure altitude caused by the rise in perigee. A third alternative is to inject at lower altitude, then keep the perigee altitude low by a sequence of thrusting maneuvers such as those shown on Figure 9. Depending on the mission profile, this last alternative may be the most advantageous one because it presents a possibility for tradeoff between the two types of ΔV requirements--requirements for high altitude departure versus requirements for maintaining low perigee altitudes.

The high inclination orbits can be used without thrusting maneuvers only if injection occurs at very high altitudes. As above, the alternative here is to inject at a reasonably low altitude and then apply several thrusts to raise perigee.

A significant feature of Figure 11 is the curve marked $i = 50^\circ$. This shows that there are parking orbits with inclinations close to 50° for which the perigee altitude remains close to 200 nautical miles for at least three months before the departure date.

As in the case of the 1975 Mars mission, the total changes over 84 days in the angular elements of two-day period parking orbits can be represented by the approximate ranges

$$-3.5^\circ < \Delta i < 0.3^\circ$$

$$-12^\circ < \Delta\Omega < 0^\circ$$

$$-3.5^\circ < \Delta\omega < 17^\circ$$

5.2 A 1976 Triple Planet Mission

One of the proposed Venus-Mars-Venus flyby missions departs earth on November 6, 1976, with a departure asymptote given by

(Reference 7)

$$\alpha_\infty = 10.99^\circ$$

$$\delta_\infty = 18.89^\circ$$

$$V_\infty = 0.2393 \text{ emos}$$

Representative perigee histories for this mission are shown in Figure 12.

The type 1, 30° orbit represents the most level curve in the family of available orbits. For two or three months before departure, this orbit calls for injection altitudes around 400 nautical miles. Extrapolating this curve backward in time shows that for a four month parking orbit, injection can be made 120 to 130 days before departure at an altitude of 200 nautical miles. During the four-month parking period, the perigee altitude will still rise to over 400 nautical miles. However, as in the case of the 1975 Venus mission, one can again apply impulsive thrusts to adjust the perigee altitude according to injection and rendezvous requirements.

Again, the 84-day changes in the angular elements are within the approximate ranges

$$-1.6^\circ < \Delta i < 1.4^\circ$$

$$-9^\circ < \Delta\Omega < 1.3^\circ$$

$$-2.5^\circ < \Delta\omega < 12^\circ$$

5.3 A 1977 Venus Flyby Mission

Figure 13 shows three perigee histories for a Venus flyby mission departing earth on January 7, 1977, with asymptote (Reference 7)

$$\alpha_\infty = 19.99^\circ$$

$$\delta_\infty = 15.51^\circ$$

$$V_\infty = 0.099 \text{ emos}$$

The two types of polar orbits have almost identical perigee histories and the type 1, 30° curve is similar to the minimum inclination ($i = \delta_\infty$) curve. Hence only one of each is plotted.

Note that the type 2, 30° orbit with a two-day period has perigee altitudes between 290 nautical miles and 140 nautical miles. The corresponding orbit with a one-day period (not shown in Figure 13) has perigee altitudes between about 215 nautical miles and 175 nautical miles. This effect of shorter orbital periods tending to "level" the curves was also shown in the case of the 1975 Mars mission. Although a shorter orbital period requires a lower injection velocity (see Figure 1) a larger impulse is required at departure time (see Figure 2).

The changes in the angular elements over 84 days of these orbits are given by the approximate ranges

$$-3^\circ < \Delta i < -0.5^\circ$$

$$-10^\circ < \Delta\Omega < 0.5^\circ$$

$$-2.5^\circ < \Delta\omega < 12.3^\circ$$

6.0 SUMMARY AND CONCLUSIONS

The effects of lunar and solar gravitation on the lifetimes of parking orbits for four selected planetary missions are shown in the form of perigee altitude histories for various orbital inclinations. Depending on solar-lunar configurations (i.e., on the dates) the perigee altitudes for some of these orbits are increasing whereas others are decreasing. Parking orbit inclinations which result in reasonably constant perigee altitudes are shown to exist at least for two Venus missions: one in mid-1975 and another in early-1977. For other orbits some thrusting will be necessary to adjust perigee altitude to meet injection and rendezvous requirements. However, the ΔV requirements were estimated to be a small portion of the overall mission requirements (see Figure 9).

For biasing and phasing the injection conditions for these four planetary missions, it is shown that orbital inclinations generally decrease; that the nodes regress; and that the location of perigee generally advances relative to the ascending node. These last two perturbations are due mostly to earth oblateness. Increasing the orbital period diminishes the effects of oblateness whereas lunar and solar perturbations become more severe.

The perturbation of perigee altitude depends on the orientation of the nodal and apsidal lines of the parking orbit relative to the position of the disturbing body at epoch (see Figure 6 and Equation 8). All of the parking orbits discussed in this memorandum are selected so that their planes contain the appropriate departure asymptote at the departure date. Thus different perigee histories may be obtained by selecting off-nominal parking orbits. This might result in severe ΔV penalties, however, because orientation or inclination changes, or both, would be required before departure and these are generally rather costly maneuvers. Of course, departing at a different time of the year would also completely change the nature of the perturbations (compare Figure 6) but the departure date is a fixed property of a particular mission rather than a free parameter.

All perigee altitudes in this memorandum are shown for 84 days and end at 200 nautical miles on the date of earth departure. This is done merely for standardization and for ease of comparison. As operational details of individual missions become available more specific computations and predictions can be made. The purpose of this memorandum is mainly to show trends and to indicate the effects of parametrization.

HB Bosch

1013-HBB-amb H. B. Bosch

Attachments
References 1-7
List of Symbols
Figures 1-13

REFERENCES

1. Moe, Mildred M.: "Solar-Lunar Perturbations of the Orbit of an Earth Satellite." ARS Journal, Volume 30, No. 5, May 1960, pp. 485-487.

2. Musen, P.; Bailie, A.; and Upton, E.: Development of the Lunar and Solar Perturbations in the Motion of an Artificial Satellite. NASA TN D-494, 1961.

3. Lorell, Jack; and Anderson, John: "Precession Rates for an Artificial Satellite." XIIIth International Astronautical Congress, Varna 1962, 1964, pp. 451-461.

4. Uphoff, C.: An Approximate Method for Predicting Lunar Satellite Lifetimes and its Application to a Lunar Orbit Mission Analysis. AAS Preprint 66-130, July 1966.

5. Wolverton, R. W. (Ed.): Flight Performance Handbook for Orbital Operations. John Wiley & Sons, Inc., 1963, Chapter 2, p. 228.

6. Ibid: Chapter 2, p. 364.

7. Personal communication from A. A. VanderVeen.

List of Symbols

Symbol

a - Semi-major axis of parking orbit

e - Eccentricity of parking orbit

h - Height above mean surface of earth

i - Inclination of parking orbit

J - Oblateness parameter (1.62345×10^{-3})

K - Constant (see Equation 8)

P - Period of parking orbit

R - Radius of earth

r - Distance from center of earth

V - Orbital velocity

V_∞ - Hyperbolic excess velocity (scalar)

$\vec{V}_\infty$ - Departure asymptote (vector)

α_∞ - Right ascension of departure asymptote

γ - Location of disturbing body at epoch (see Figure 6)

Δ - Change in the associated variable

ΔV_H - Characteristic velocity for Hohmann transfer

ΔV_i - Characteristic velocity for three-impulse injection maneuver

ΔV_o - Characteristic velocity for two-impulse perigee adjusting maneuver

δ_∞ - Declination of departure asymptote

θ_∞ - Angle from perigee to departure asymptote

μ - Gravitational constant of earth

Ω - Right ascension of ascending node

ω - Argument of perigee

List of Symbols (Cont'd)

Subscript

a - Apogee of parking orbit

c - Circular orbit

L - Perigee of earth escape hyperbola

p - Perigee of parking orbit

r - Relative to orbit of disturbing body

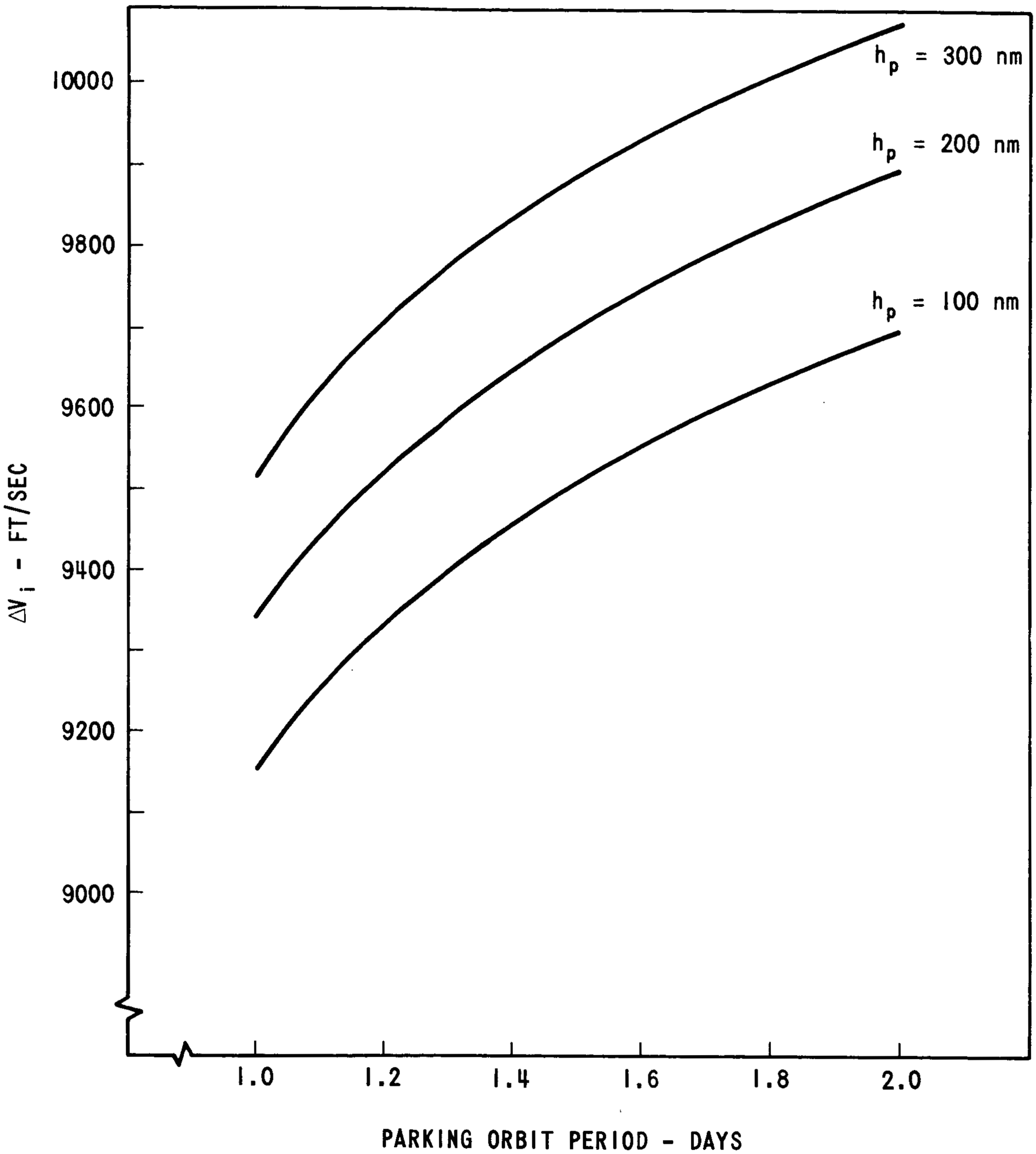

FIGURE 1 - CHARACTERISTIC VELOCITY (ΔV_i) REQUIREMENTS FOR INJECTION INTO ELLIPTIC PARKING ORBIT FROM CIRCULAR ORBIT OF 100 NM ALTITUDE

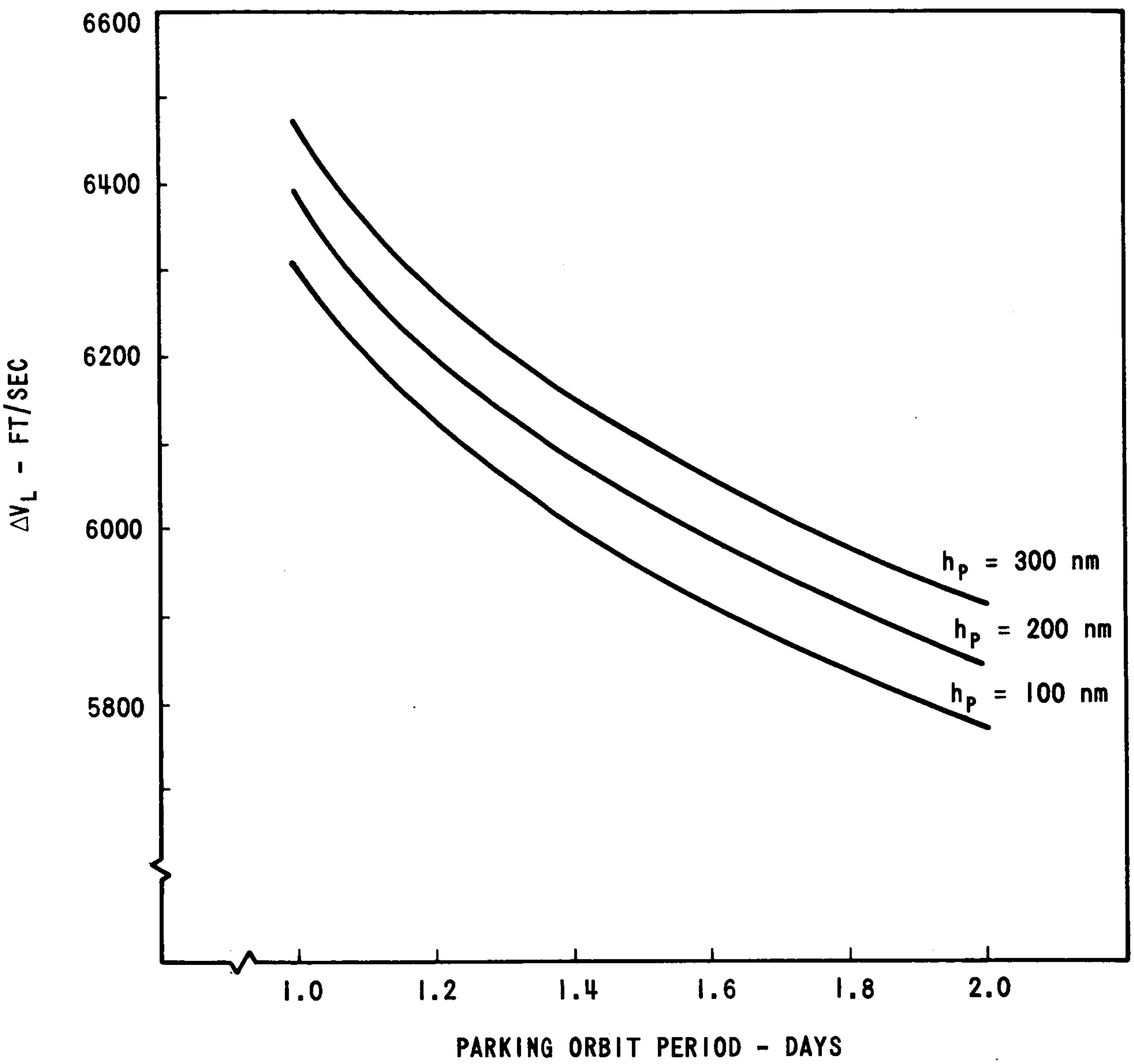

FIGURE 2 - CHARACTERISTIC VELOCITY (ΔV_L) REQUIREMENTS TO ATTAIN HYPERBOLIC EXCESS SPEED OF V_∞ = 0.1985 EMOS FROM PERIGEE OF ELLIPTIC PARKING ORBIT

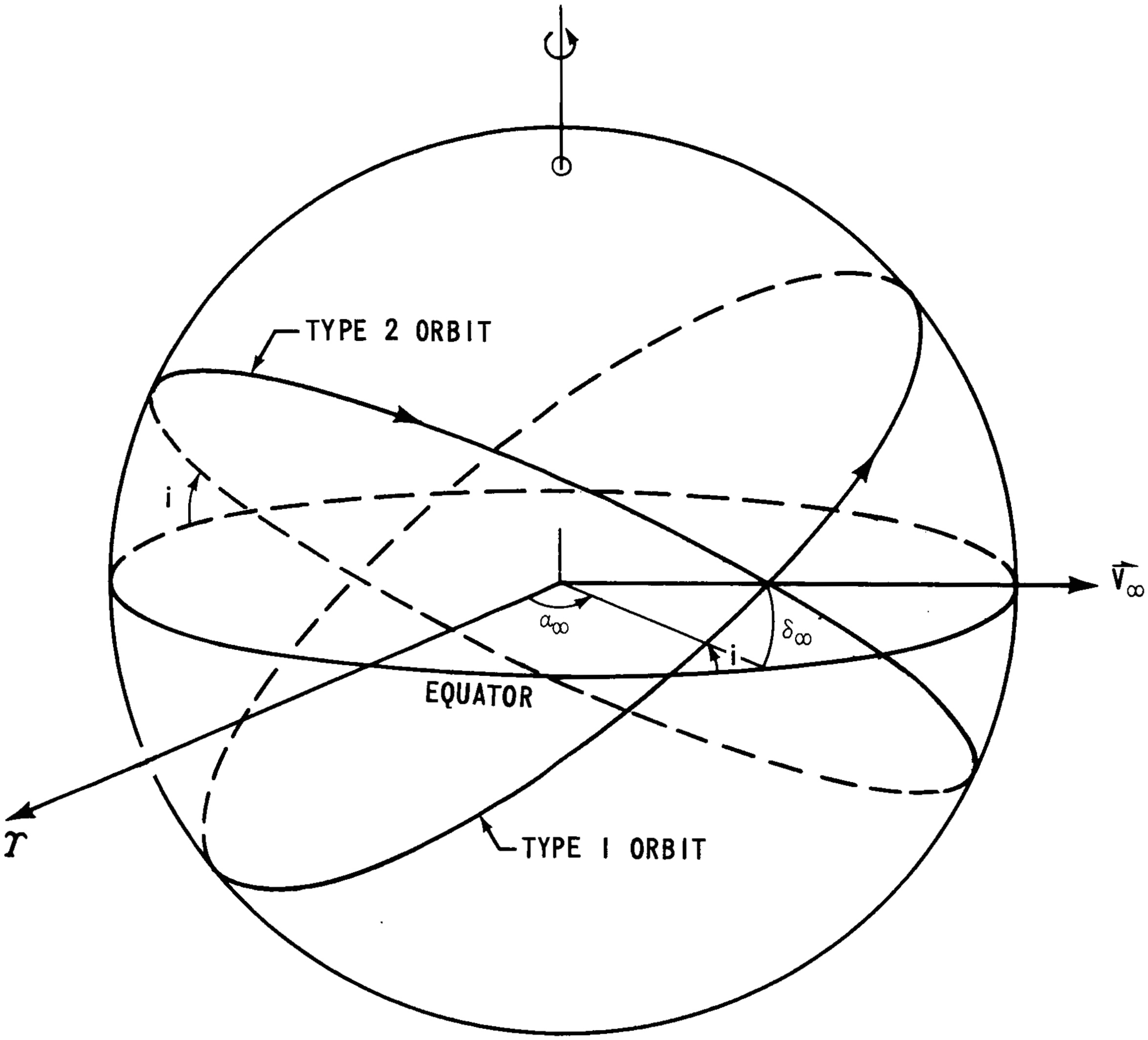

FIGURE 3 - PROJECTIONS ON THE CELESTIAL SPHERE OF THE TWO TYPES OF PARKING ORBITS FOR A GIVEN DEPARTURE ASYMPTOTE ($\vec{V}_\infty$)

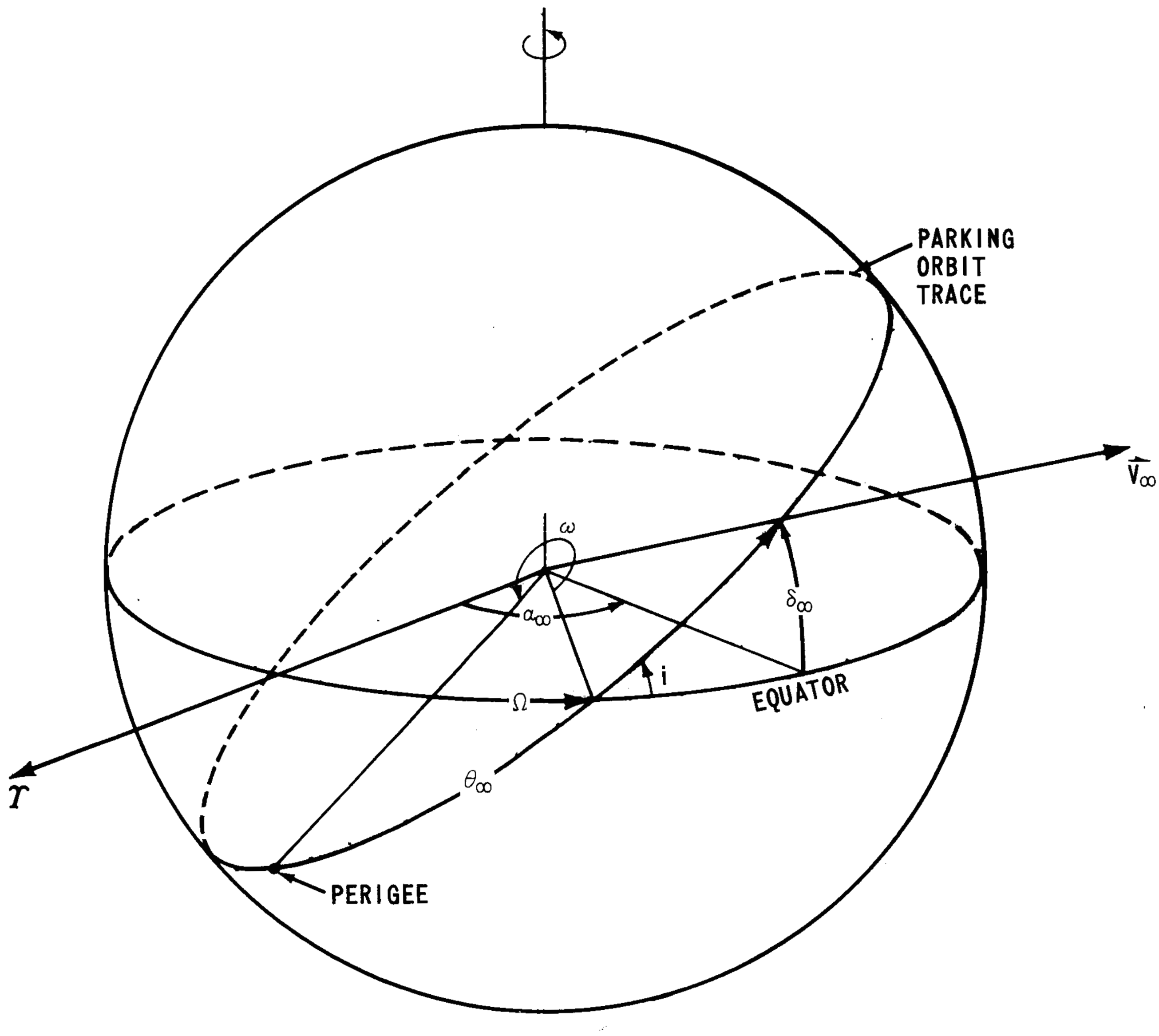

FIGURE 4 - PARKING ORBIT PARAMETERS AS RELATED TO THE DEPARTURE ASYMPTOTE ($\vec{V}_\infty$)

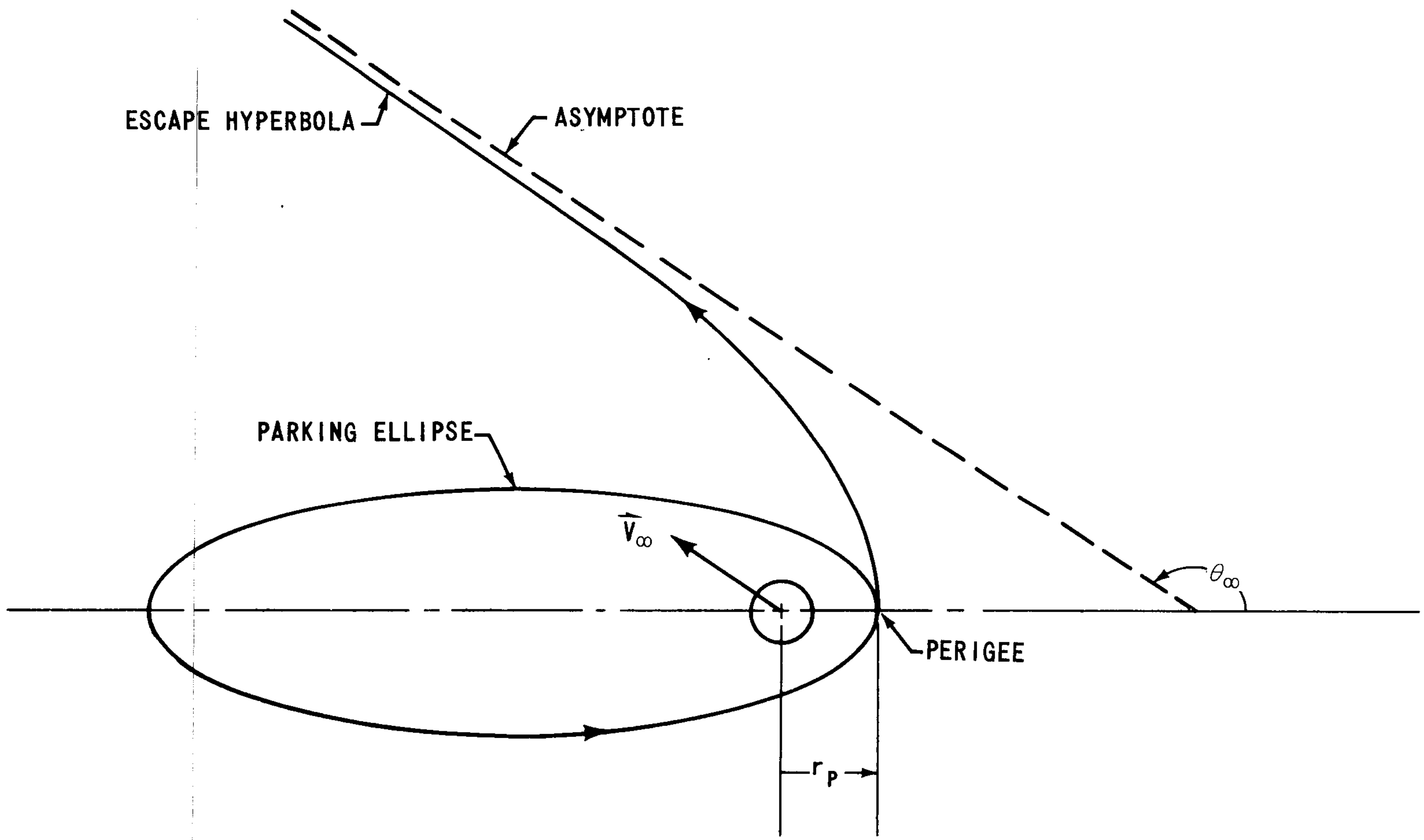

FIGURE 5 - RELATIONSHIP BETWEEN PARKING ELLIPSE, ESCAPE HYPERBOLA AND DEPARTURE ASYMPTOTE

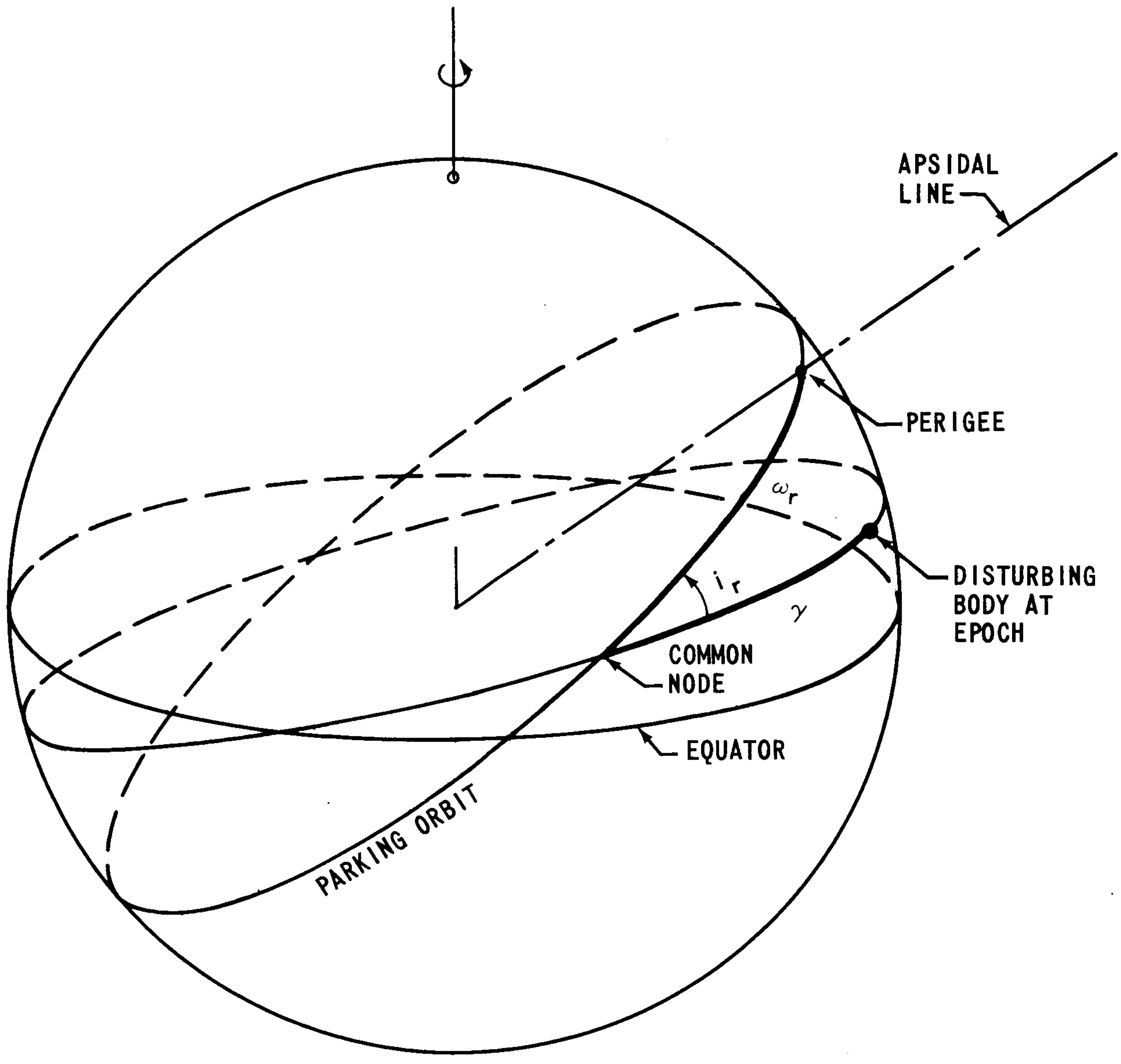

FIGURE 6 - PROJECTION ON THE CELESTIAL SPHERE SHOWING ORIENTATION OF PARKING ORBIT RELATIVE TO ORBIT OF DISTURBING BODY (SUN OR MOON)

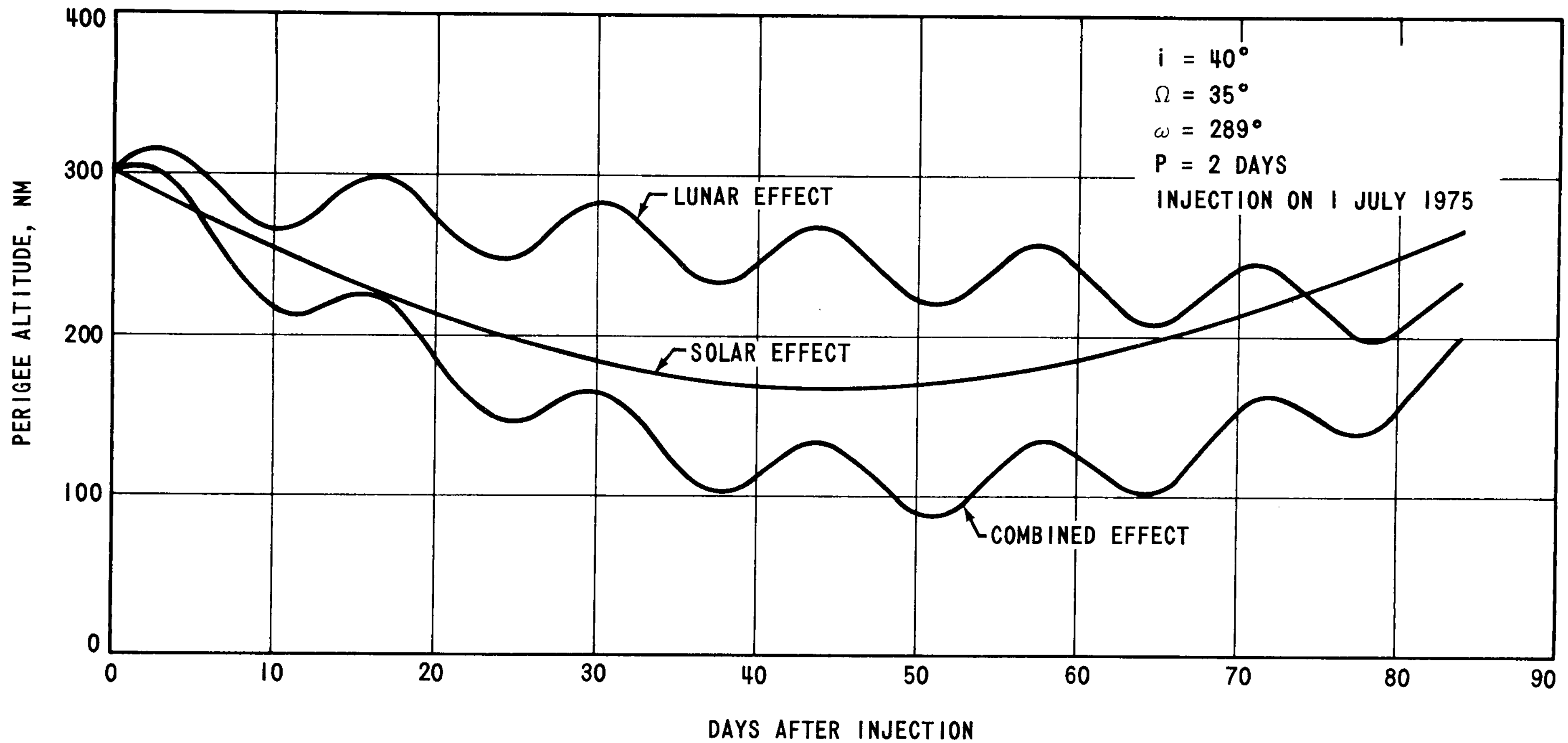

FIGURE 7 - EFFECTS OF SUN AND MOON ON PERIGEE ALTITUDE OF ELLIPTIC EARTH ORBIT

DEPARTURE DATE: 23 SEPTEMBER 1975

DEPARTURE ASYMPTOTE $\begin{cases} \alpha_\infty = 85.7° \\ \delta_\infty = 32.95° \\ V_\infty = 0.1985 \text{ EMOS} \end{cases}$

ORBITAL PERIOD: 2 DAYS

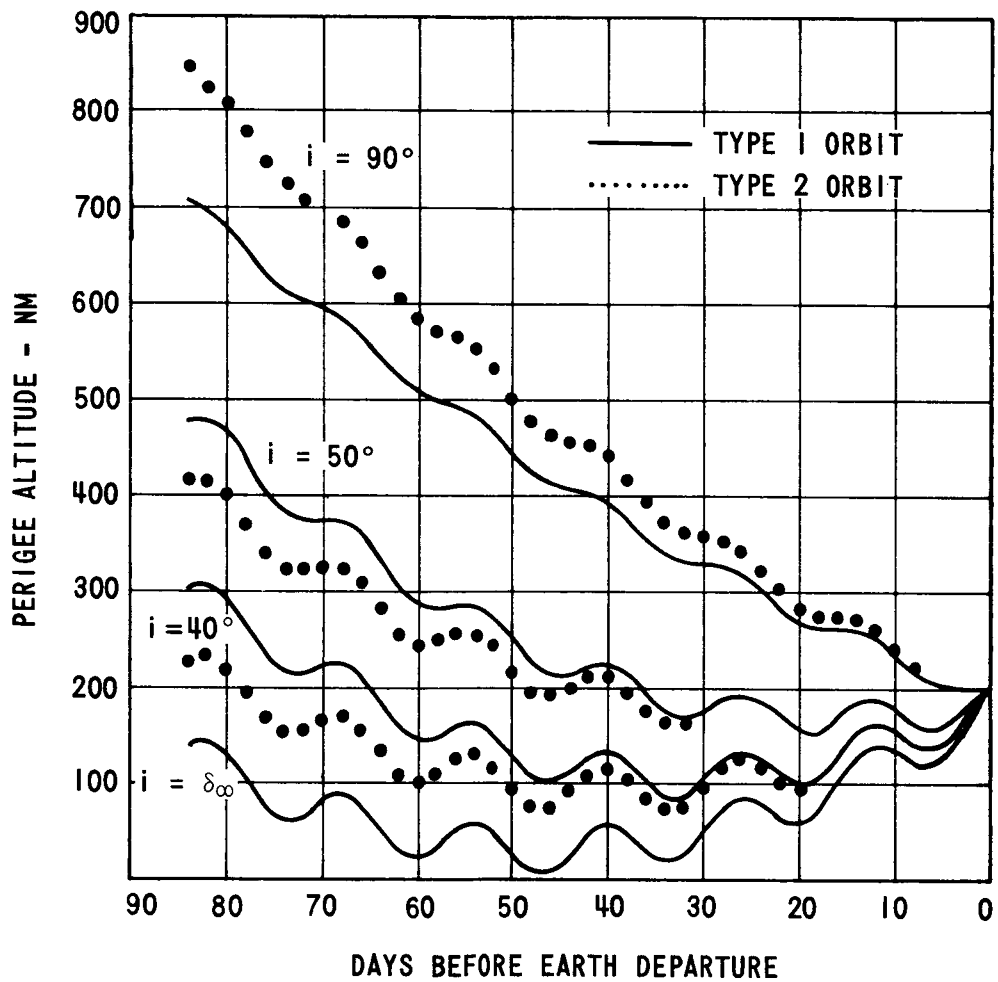

FIGURE 8a - EFFECT OF ORBITAL INCLINATION ON PERIGEE ALTITUDE OF EARTH PARKING ORBITS FOR A MARS FLYBY MISSION

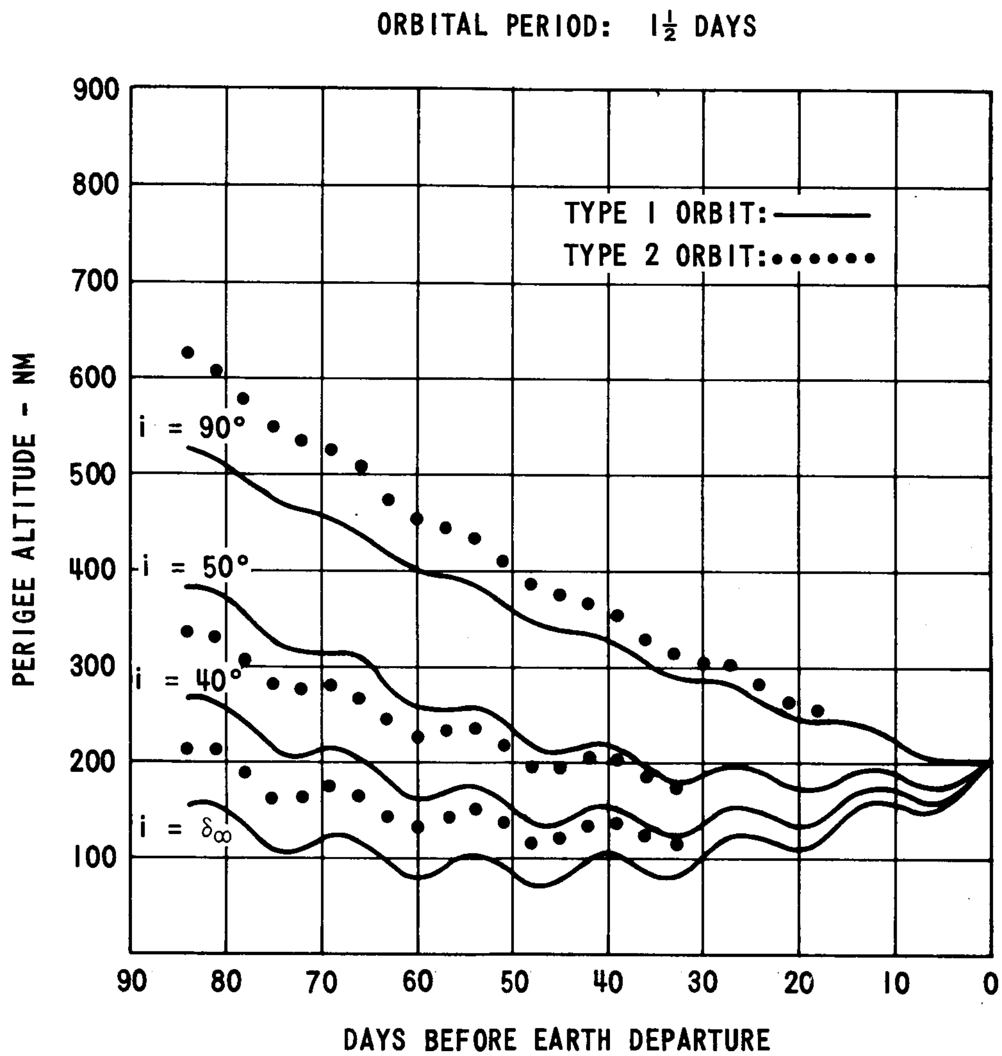

FIGURE 8b - EFFECT OF ORBITAL INCLINATION ON PERIGEE ALTITUDE OF EARTH PARKING ORBITS FOR A MARS FLYBY MISSION (SEE FIGURE 8a)

ORBITAL PERIOD: I DAY

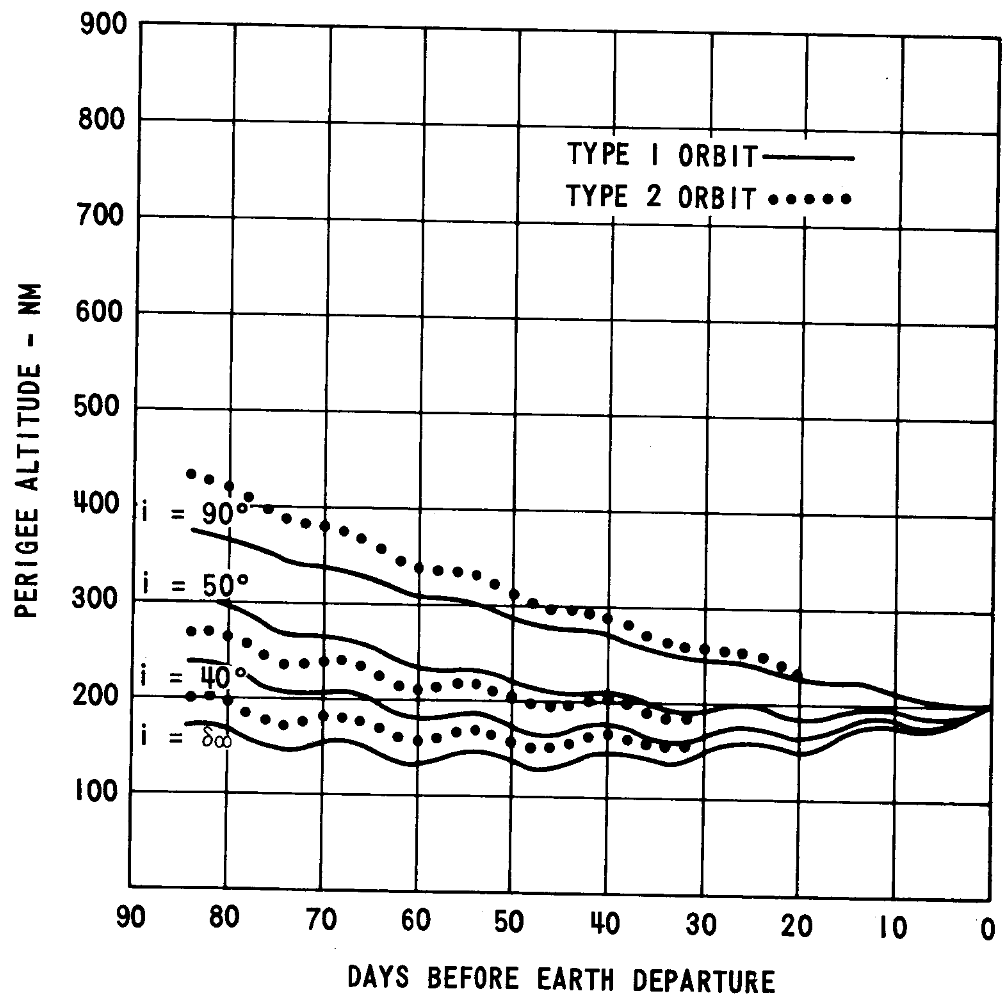

FIGURE 8c - EFFECT OF ORBITAL INCLINATION ON PERIGEE ALTITUDE OF EARTH PARKING ORBITS FOR A MARS FLYBY MISSION (SEE FIGURE 8a)

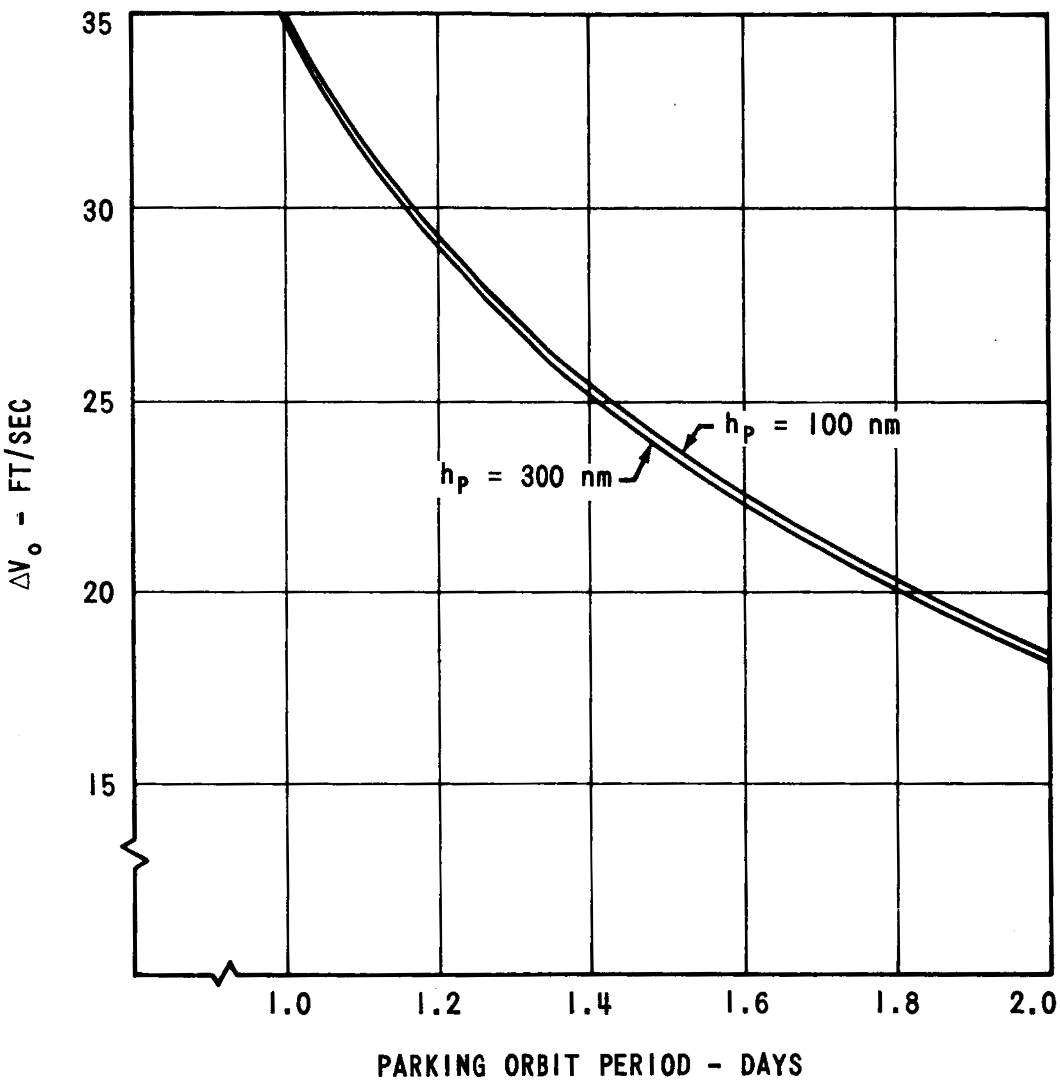

FIGURE 9 - CHARACTERISTIC VELOCITIES (ΔV_o) REQUIRED TO RAISE PERIGEE ALTITUDE BY 50 NM WHILE MAINTAINING CONSTANT ORBITAL PERIOD

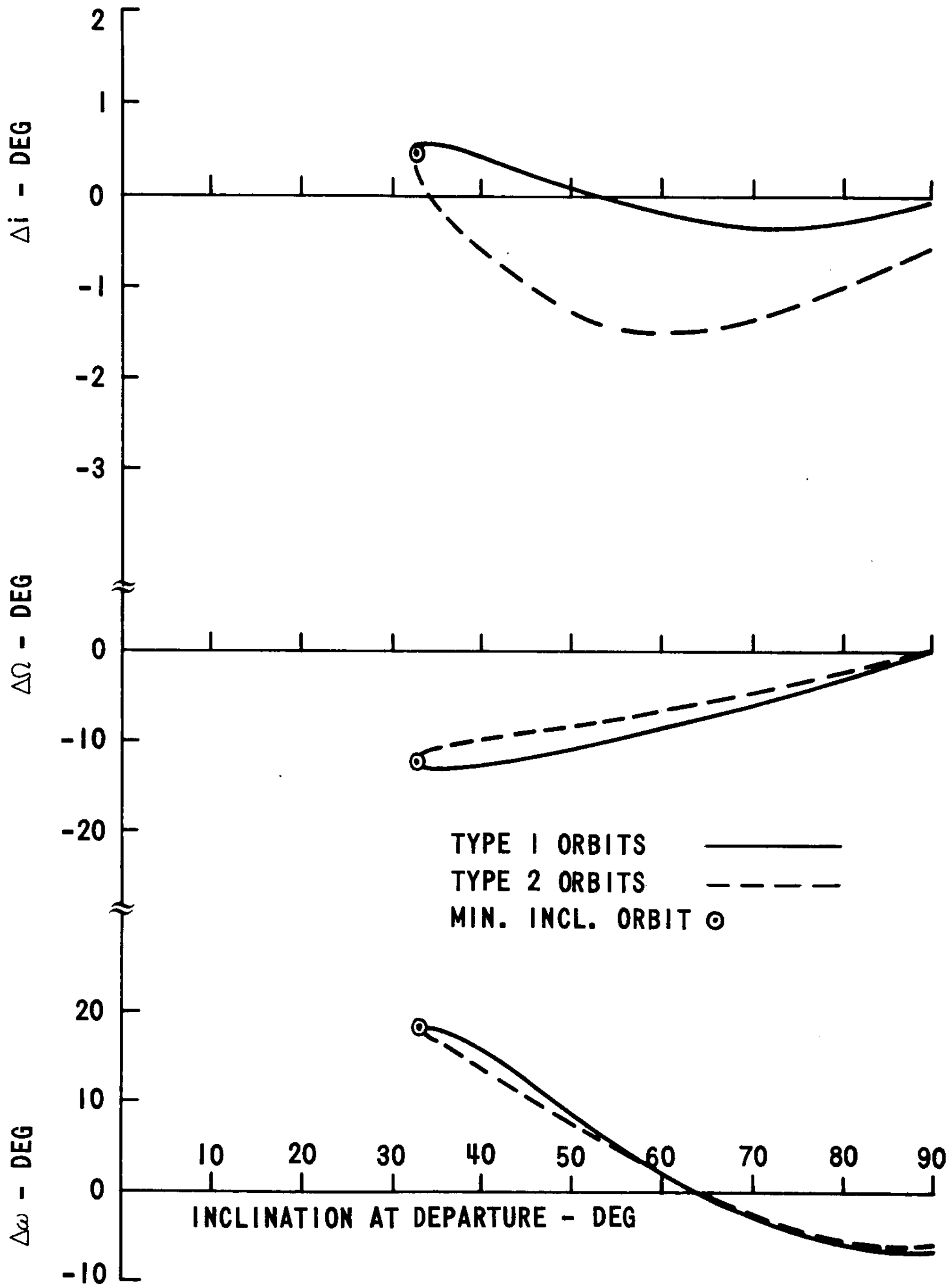

FIGURE 10a - TOTAL CHANGES OVER 84 DAYS IN THE INCLINATION (Δi), NODE ($\Delta\Omega$), AND ARGUMENT OF PERIGEE ($\Delta\omega$) OF PARKING ORBITS FOR A 1975 MARS MISSION - ONE DAY ORBITAL PERIOD

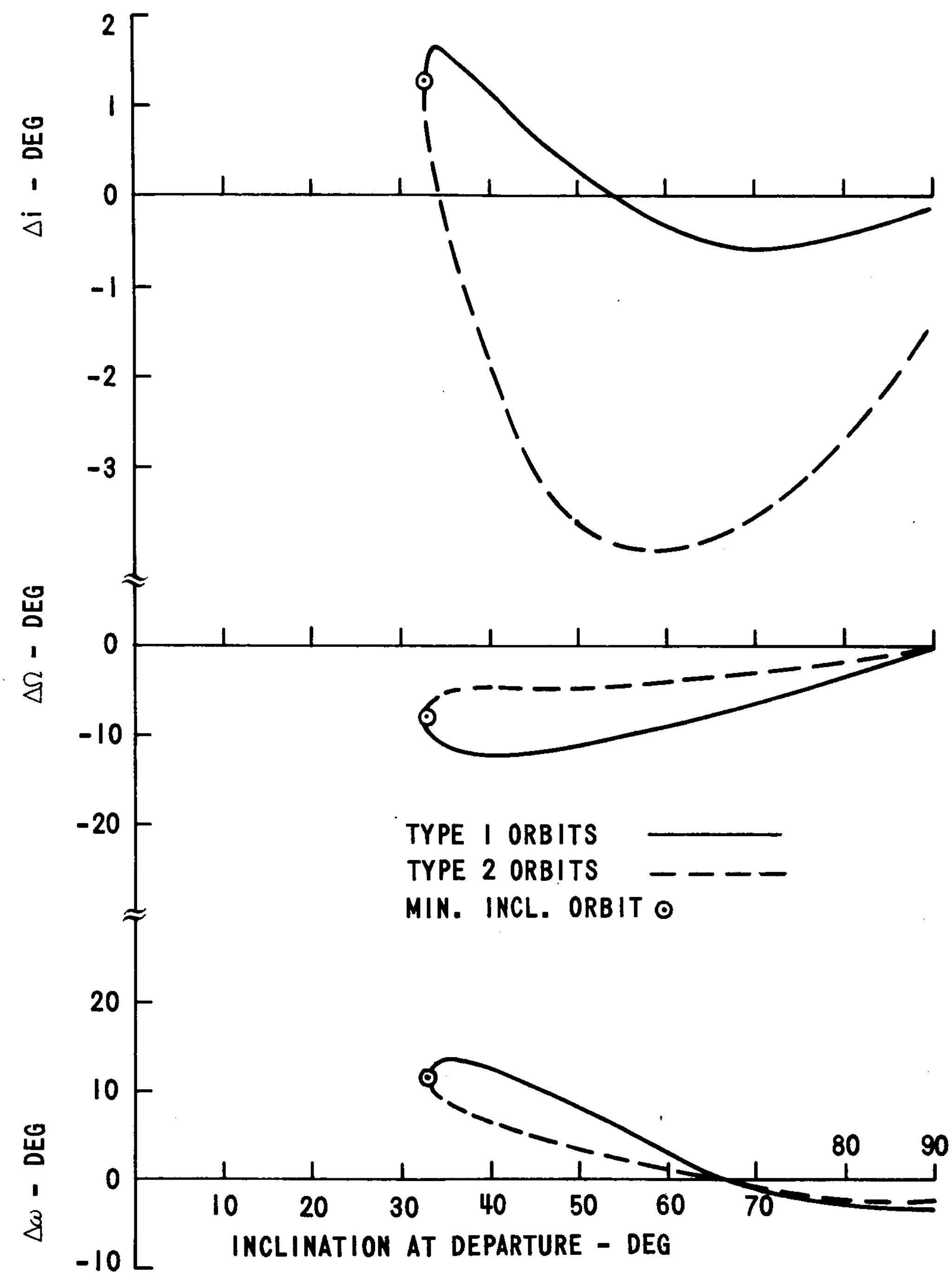

FIGURE 10b - TOTAL CHANGES OVER 84 DAYS IN THE INCLINATION (Δi), NODE ($\Delta\Omega$), AND ARGUMENT OF PERIGEE ($\Delta\omega$) OF PARKING ORBITS FOR A 1975 MARS MISSION - TWO DAY ORBITAL PERIOD

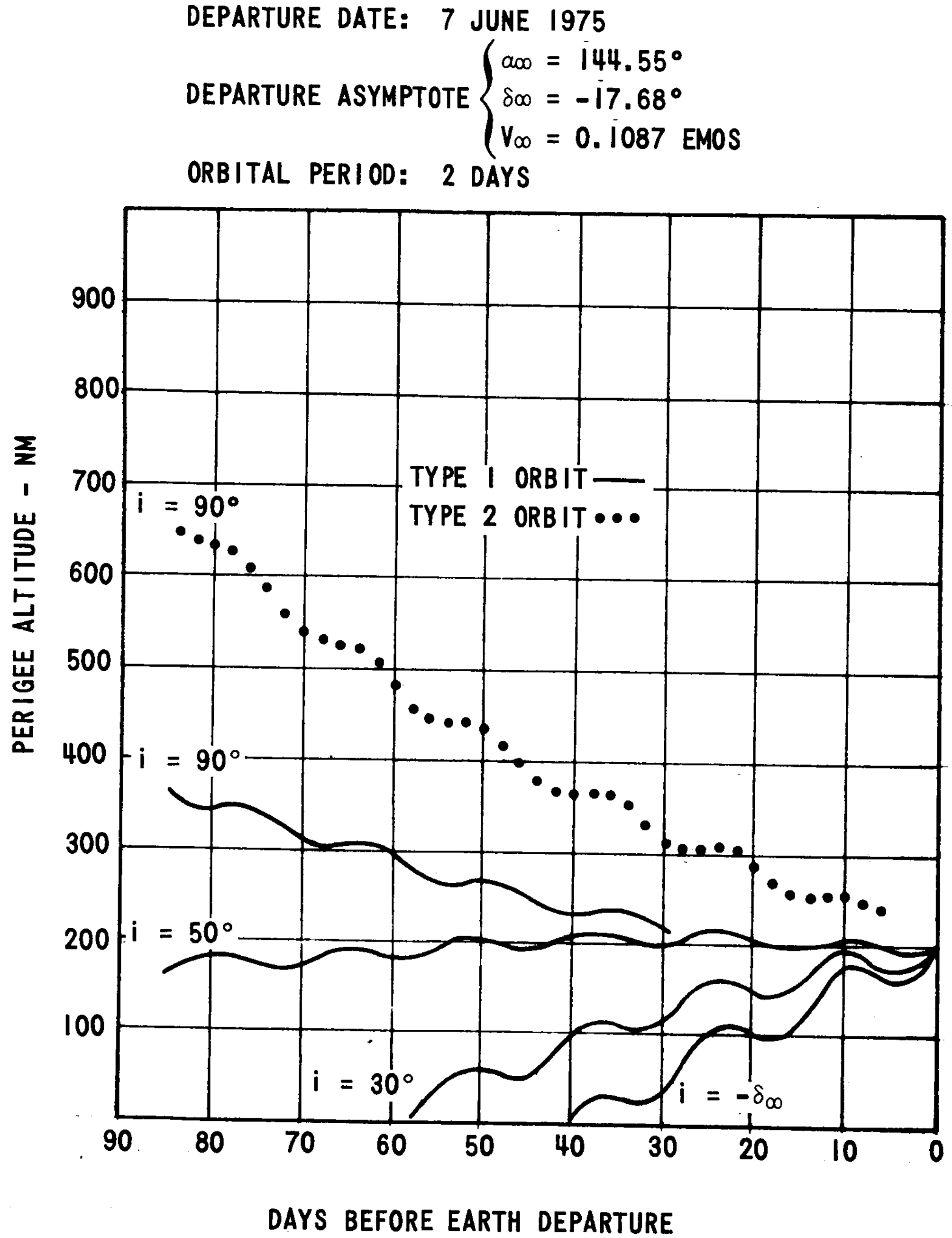

FIGURE 11 - EFFECT OF ORBITAL INCLINATION ON PERIGEE ALTITUDE OF EARTH PARKING ORBITS FOR A VENUS FLYBY MISSION

DEPARTURE DATE: 6 NOVEMBER 1976

DEPARTURE ASYMPTOTE $\begin{cases} \alpha_\infty = 10.99° \\ \delta_\infty = 18.89° \\ V_\infty = 0.2393 \text{ EMOS} \end{cases}$

ORBITAL PERIOD: 2 DAYS

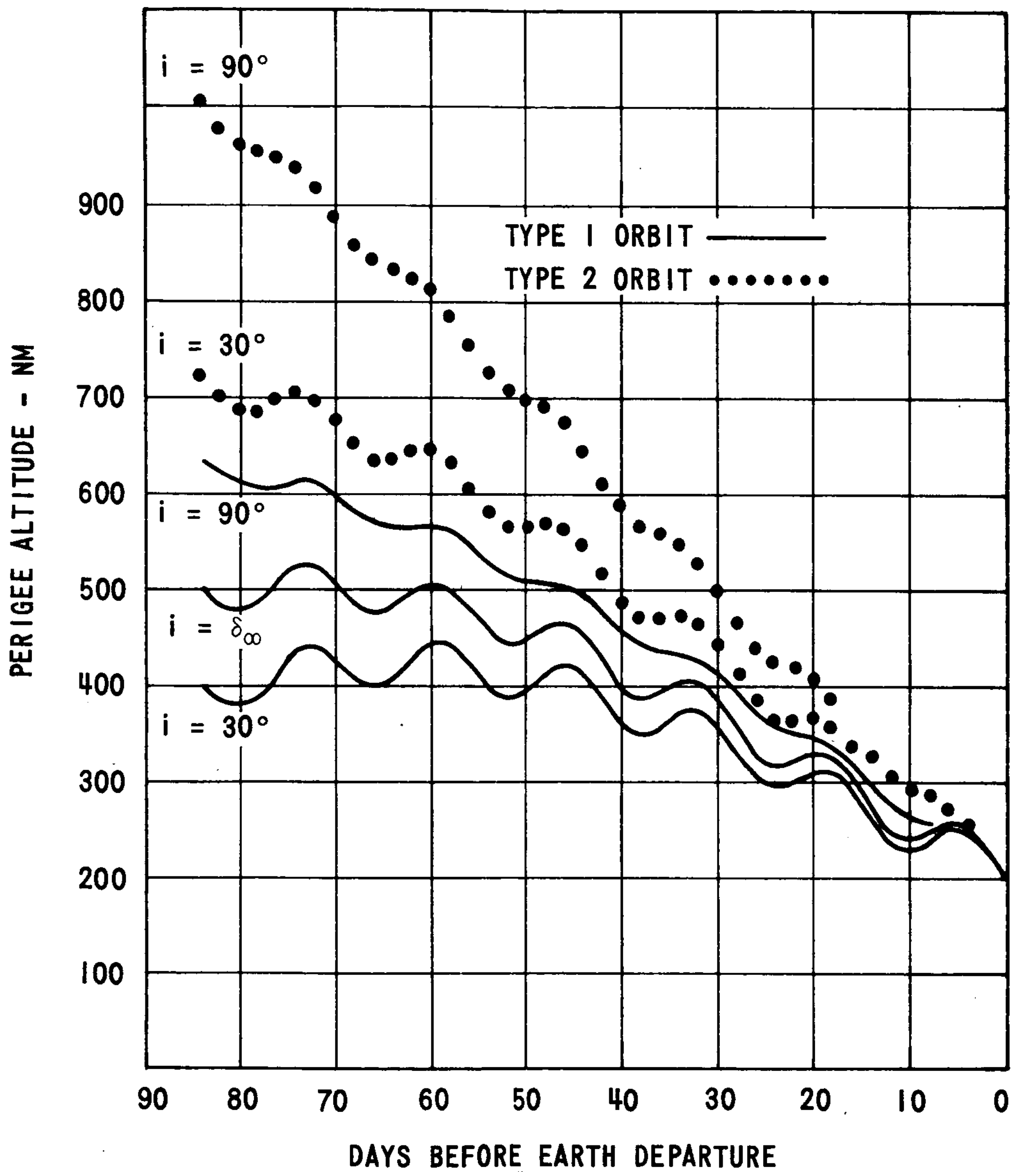

FIGURE 12 - EFFECT OF ORBITAL INCLINATION ON PERIGEE ALTITUDE OF EARTH PARKING ORBITS FOR A TRIPLE PLANET MISSION

DEPARTURE DATE: 7 JANUARY 1977

DEPARTURE ASYMPTOTE $\begin{cases} \alpha_\infty = 19.99° \\ \delta_\infty = 15.51° \\ V_\infty = 0.099 \text{ EMOS} \end{cases}$

ORBITAL PERIOD: 2 DAYS

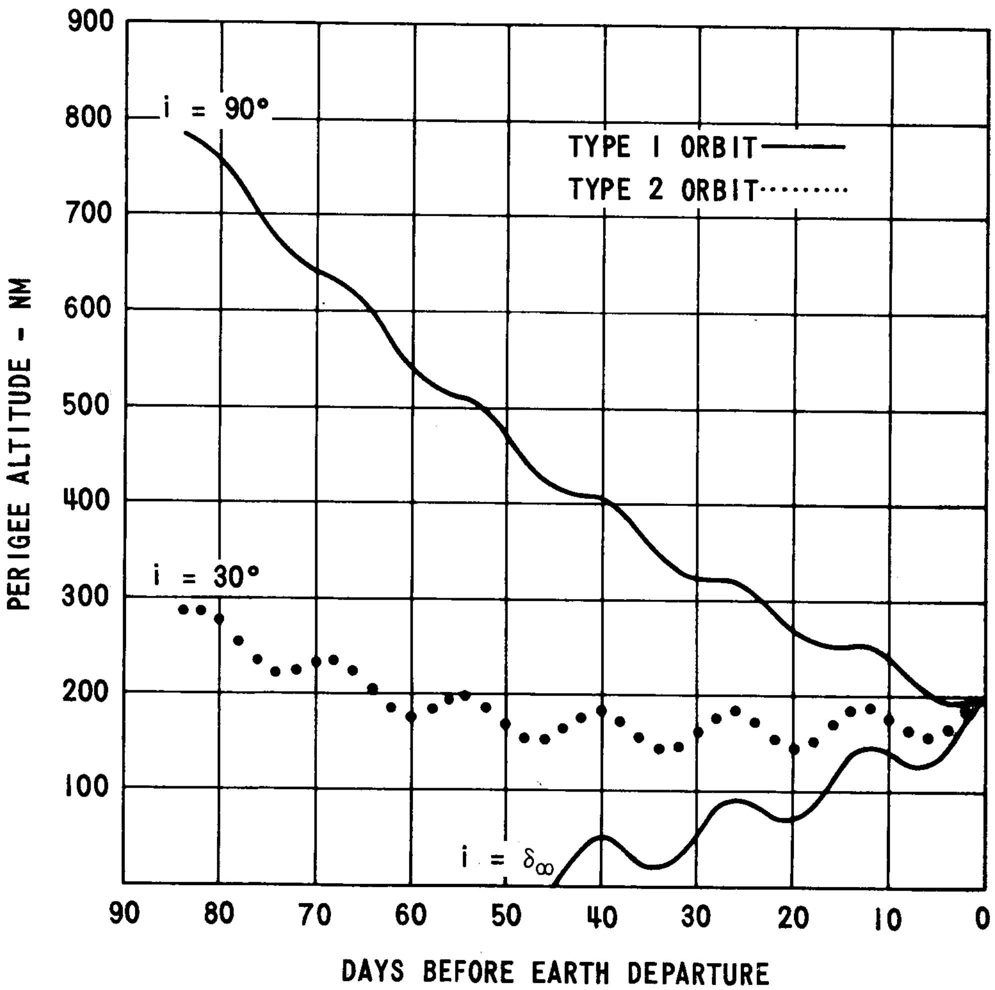

FIGURE 13 - EFFECT OF ORBITAL INCLINATION ON PERIGEE ALTITUDE OF EARTH PARKING ORBITS FOR A VENUS FLYBY MISSION